Peak-Background Split and Primordial non-Gaussianity

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with Vincent Desjacques, Donghui Jeong,
Marc Kamionkowski

Michigan Non-Gaussianity workshop, 5/14/2011

Motivation

- Large-scale structure (LSS) expected to give *most stringent constraints* on non-Gaussianity (NG) – in the long run...

- In order use LSS tracers, we *have to understand biasing* in non-Gaussian case.

I. Describing Non-Gaussianity

- Either via *bispectrum*

$$\langle\hat{\phi}(\vec{k}_1)\hat{\phi}(\vec{k}_2)\hat{\phi}(\vec{k}_3)\rangle = (2\pi)^3\delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)B_\phi(k_1,k_2,k_3)$$

Bardeen potential during mat. dom.

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- Or via *field redefinition*

$$\hat{\phi}(\vec{k}) = \phi(\vec{k}) + f_{\rm NL}\int \frac{d^3\vec{k}_1}{(2\pi)^3}\,\omega(\vec{k}_1,\vec{k}-\vec{k}_1)\phi(\vec{k}_1)\phi(\vec{k}-\vec{k}_1)$$

Gaussian random field

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  – Used for N-body simulations

I. Describing Non-Gaussianity

- Relation to bispectrum:

$$B_\phi(k_1, k_2, k_3) = 2 f_{\rm NL} \omega(\vec{k}_1, \vec{k}_2) P_\phi(k_1) P_\phi(k_2) + \text{perm.}$$

  - Note: *bispectrum does not specify ω uniquely*
  - Results on large scales indep. of kernel choice
  - One possible choice:

$$\omega(k_1, k_2, k_3) = \frac{1}{2 f_{\rm NL}} \frac{B_\phi(k_1, k_2, k_3)}{P_{\phi 1} P_{\phi 2} + P_{\phi 1} P_{\phi 3} + P_{\phi 2} P_{\phi 3}}$$

Peak-Background Split (PBS)

- Write perturbations as: $\delta = \delta_l + \delta_s,\ \phi = \phi_l + \phi_s,\ \ldots$

Scales which govern halo formation

Scales on which clustering is measured

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See also Vincent's talk for complementary approach (conditional mass function)

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- Throughout, assume universal mass function:

$$n_h = \frac{dn}{d\ln M} = \frac{\bar{\rho}}{M} f(\nu) \frac{\partial \ln \nu}{\partial \ln M}, \qquad \nu = \delta_c / \sigma_M$$

Variance of δ on scale *M*

Halo Bias in PBS

- Large-scale δ changes collapse threshold:

$$\delta_c \to \delta_c - \delta_l \quad \Rightarrow \quad b_1 = -\frac{1}{\sigma_M}\frac{\partial \ln n_h}{\partial \nu}$$

Mo & White 96

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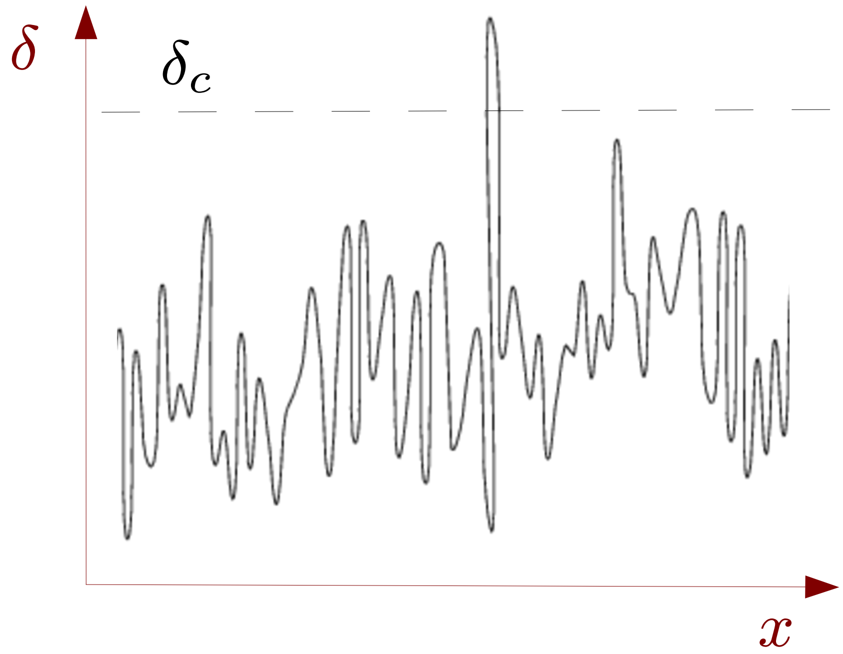

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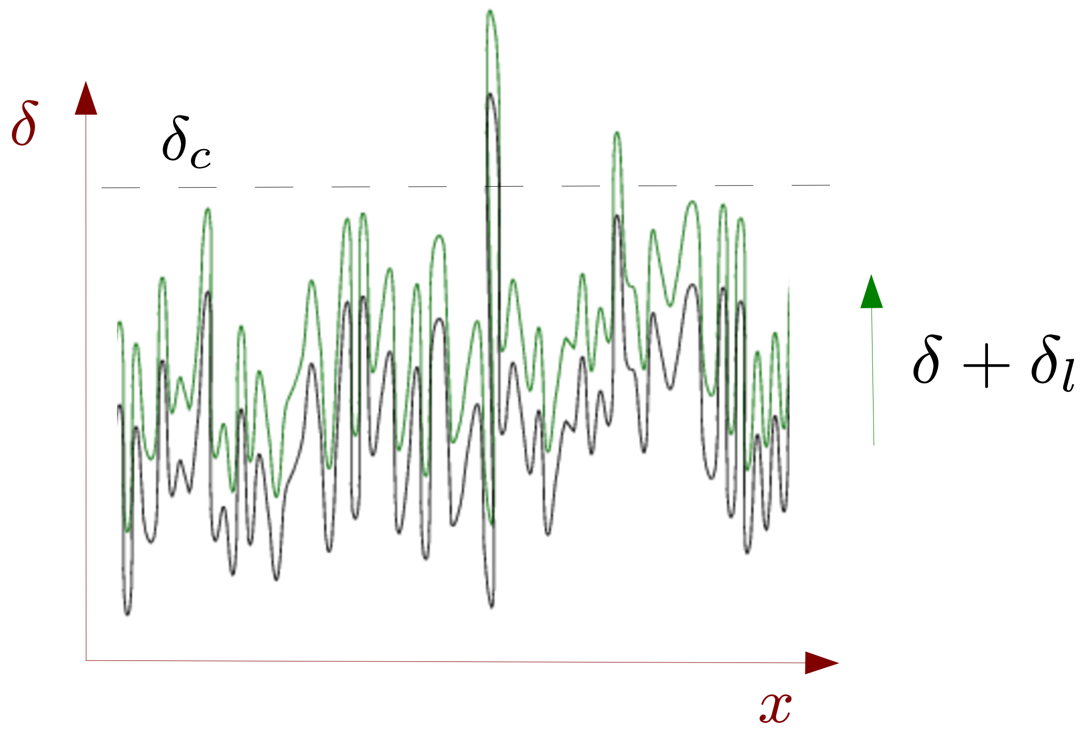

NG halo bias

- Apply Poisson equation to $\hat{\phi}(k) = \phi(k) + f_{\rm NL} \int \omega\, \phi\, \phi$
  - l-s-split... only retain mixed l-s terms

$$\Rightarrow \hat{\delta}_s = \delta_s + f_{\rm NL} \int \omega\, \phi_l\, \delta_s$$

FS & Kamionkowski

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$$\hat{\sigma}^2_M = \sigma^2_M + 4 f_{\rm NL} \sigma^2_{\omega M}(k) \phi_l(k)$$

(for single long-wavelength mode)

$$\sigma^2_{\omega M}(k) \equiv \int \frac{d^3 k_s}{(2\pi)^3}\, \omega(\vec{k}, \vec{k}_s) W^2_M(k_s) P(k_s)$$

Tophat filter of scale *M*

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Tophat filter of scale *M*

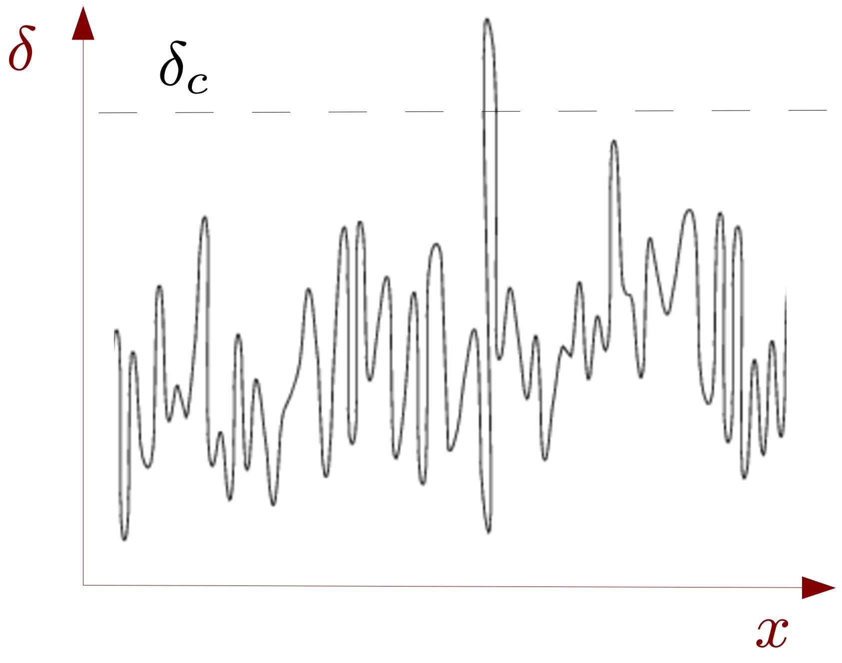
  - Local fNL: $\omega(\vec{k}, \vec{k}_s) = 1$

$$\Rightarrow \hat{\sigma}^2_M = \sigma^2_M (1 + 4 f_{\rm NL} \phi_l)$$

Dalal et al,
Slosar et al

Non-Gaussianity in PBS

$$\hat{\sigma}_M^2 = \sigma_M^2 + 4 f_{\mathrm{NL}} \sigma_{\omega M}^2(k)\phi_l(k)$$



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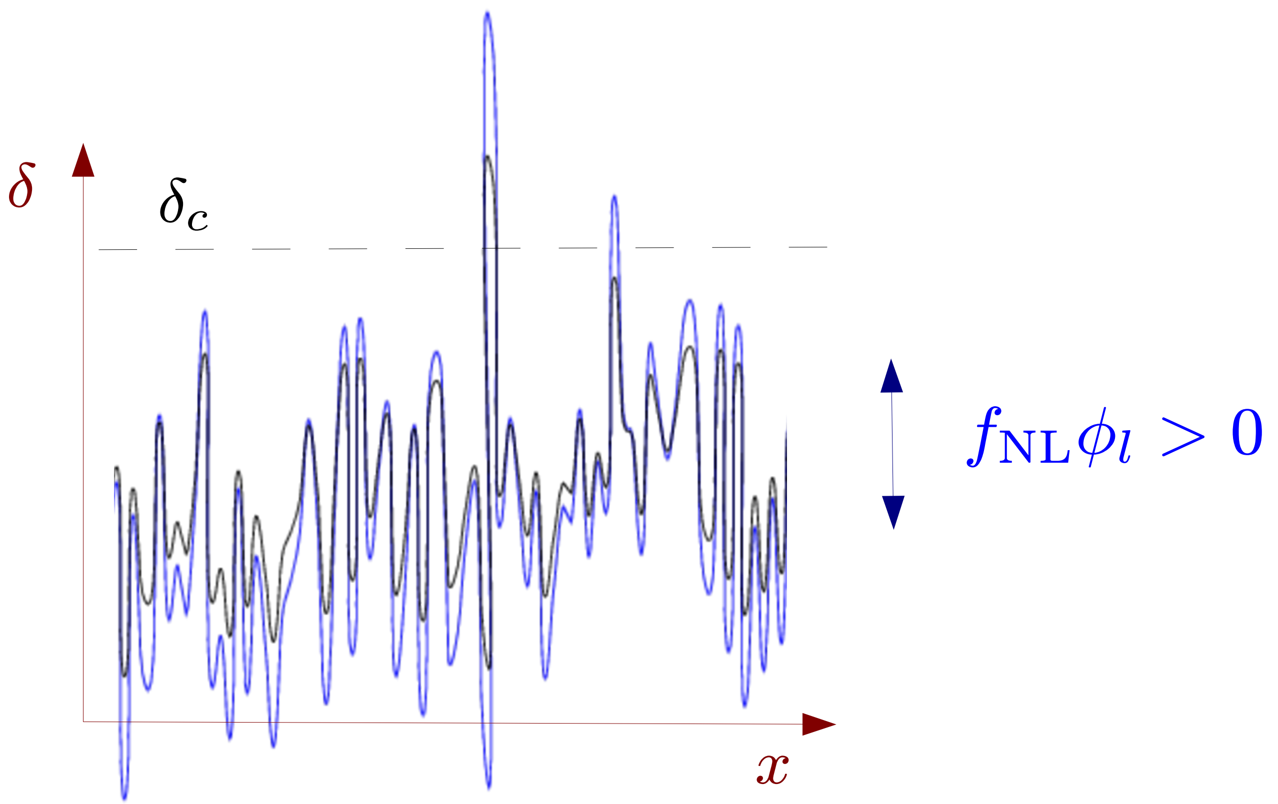

NG halo bias

- Apply chain rule:

$$\Delta b_1(k) = \frac{\partial \ln n_h}{\partial \ln \hat{\sigma}_M} \frac{\partial \ln \hat{\sigma}_M}{\partial \ln \phi_l(k)} \frac{\partial \ln \phi_l(k)}{\partial \ln \delta_l(k)}$$

FS & Kamionkowski

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From universal mass function, and $\nu = \delta_c/\sigma_M$

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$$= 2 f_{\rm NL} \mathcal{M}^{-1}(k)\, b_1 \delta_c \, \frac{\sigma^2_{\omega M}(k)}{\sigma^2_M}$$

$$\mathcal{M}(k) = \frac{2}{3} \frac{k^2 T(k) g(z)}{\Omega_m H_0^2 (1+z)}$$

FS & Kamionkowski

Compare with Local Biasing

- Assume halo density is *local function* of linear matter density:

Fry & Gaztanaga 93

$$n_h(\vec{x}) = \bar{n}_h \cdot \left(1 + b_1 \delta(\vec{x}) + \frac{b_2}{2}\delta^2(\vec{x}) + ...\right)$$

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- Halo power spectrum with NG:

$$P_h(k) = b_1^2 P(k) + b_1\, b_2\, \mathcal{P}(k)$$

$$\mathcal{P}(k) = \int \frac{d^3k_1}{(2\pi)^3} B_m(k_1, |\vec{k}_1 - \vec{k}|, k)$$

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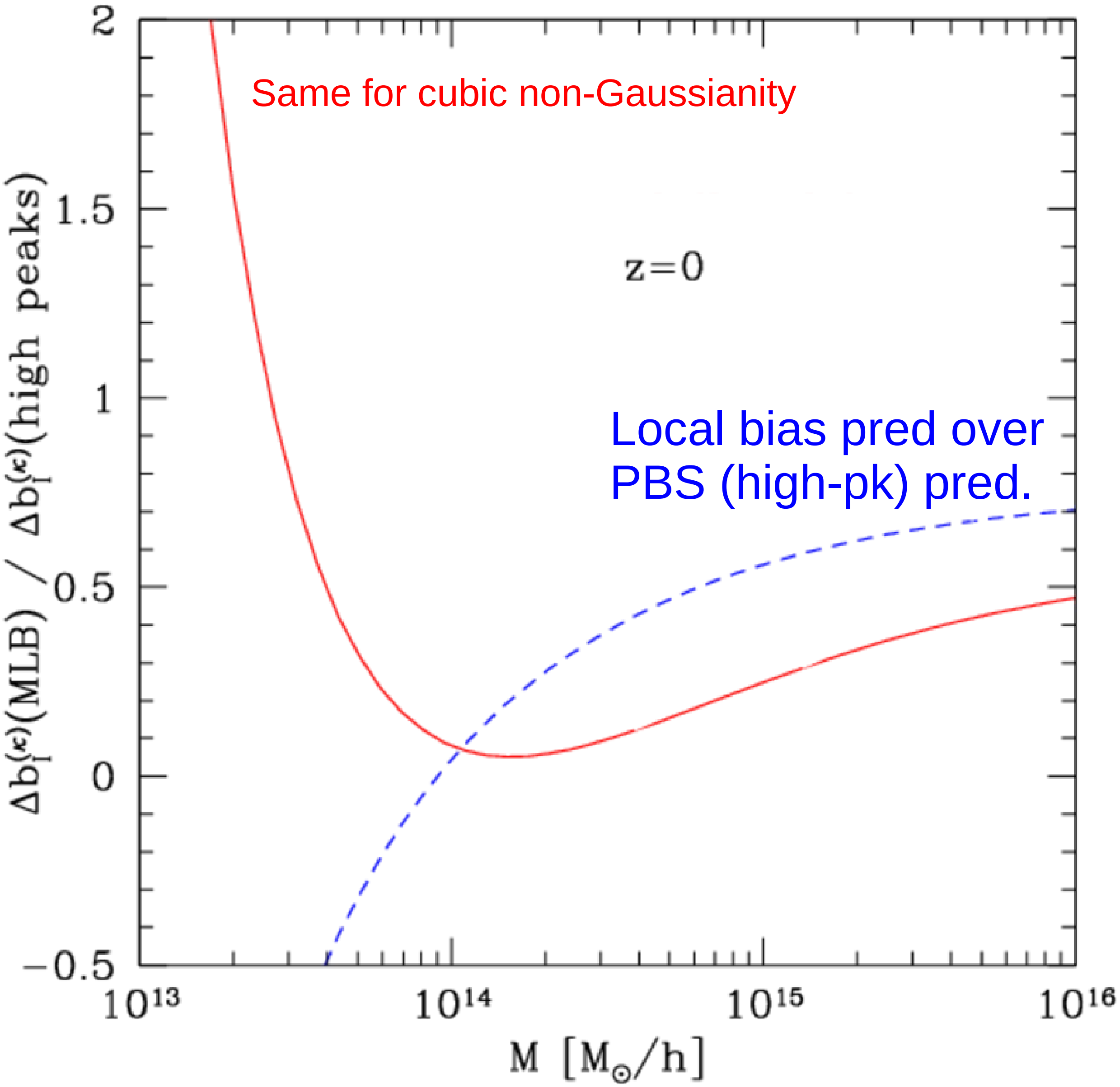
- PBS and local bias agree in *high-peak limit* (of PS theory) where

$$b_2 = b_1^2 = \frac{\nu^2}{\sigma^2_M}$$

High-peak limit vs real halos



Desjacques, Jeong, FS, in prep

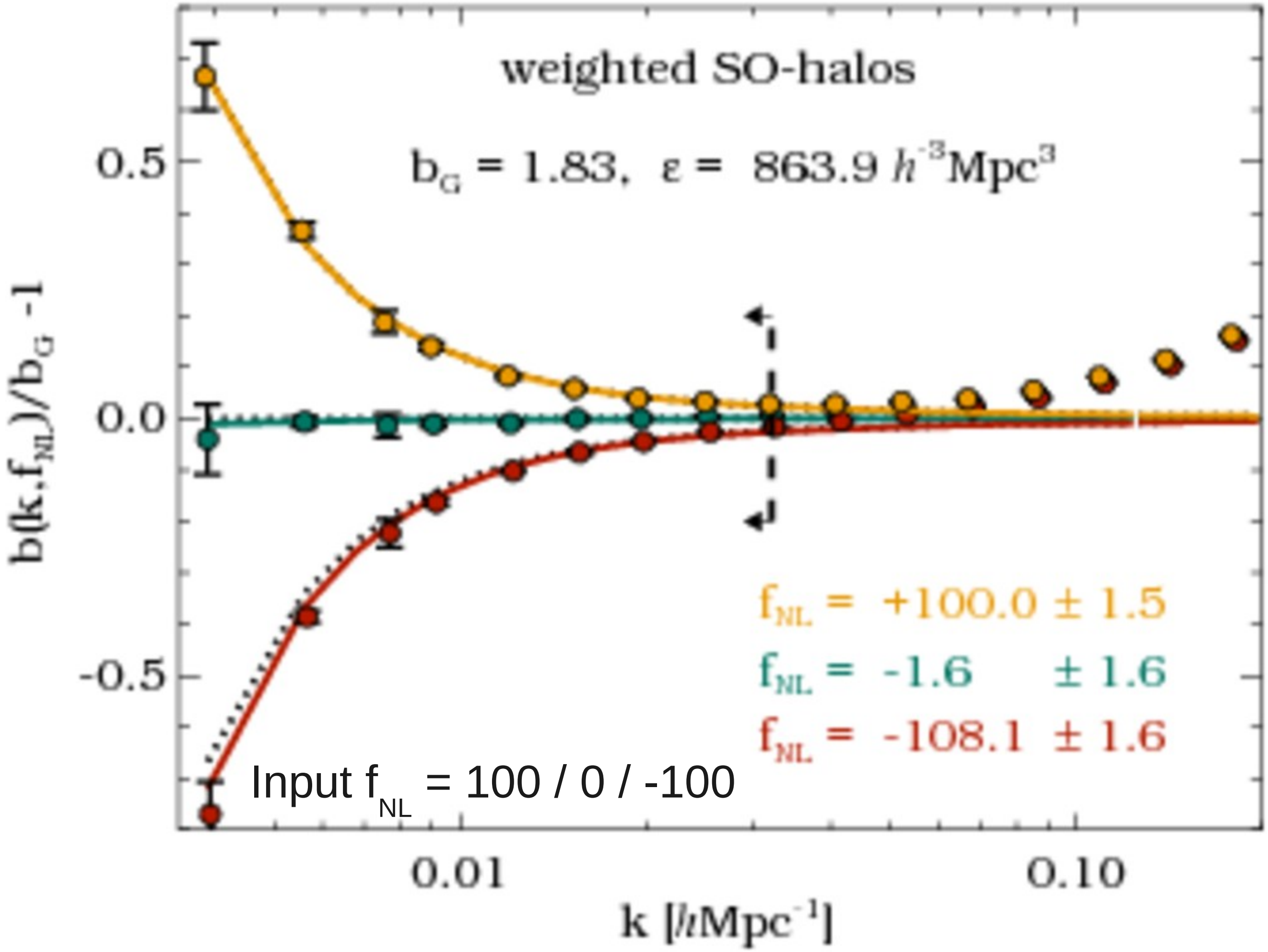
Predictions for Δb

- Scaling of Δb(k) determined by *squeezed limit* of non-Gaussian kernel: $\omega(k_s, k, |\vec{k} - \vec{k}_s|),\ k_s \gg k$
  - *Local model:* $\omega \to 1 \Rightarrow \sigma_\omega^2 = \sigma_M^2 \Rightarrow \Delta b_1 \propto k^{-2}$

    Scale-dep. local: $\Delta b_1 \propto k^{-2}$ with different amplitude
  - *Equilateral model:* $\omega \propto k^2 \Rightarrow \Delta b_1 \approx \text{const.}$
  - *Folded model:* $\omega \propto k \Rightarrow \Delta b_1 \propto k^{-1}$

II. Comparison with N-body results

- Local f_{NL} ... PBS prediction works !



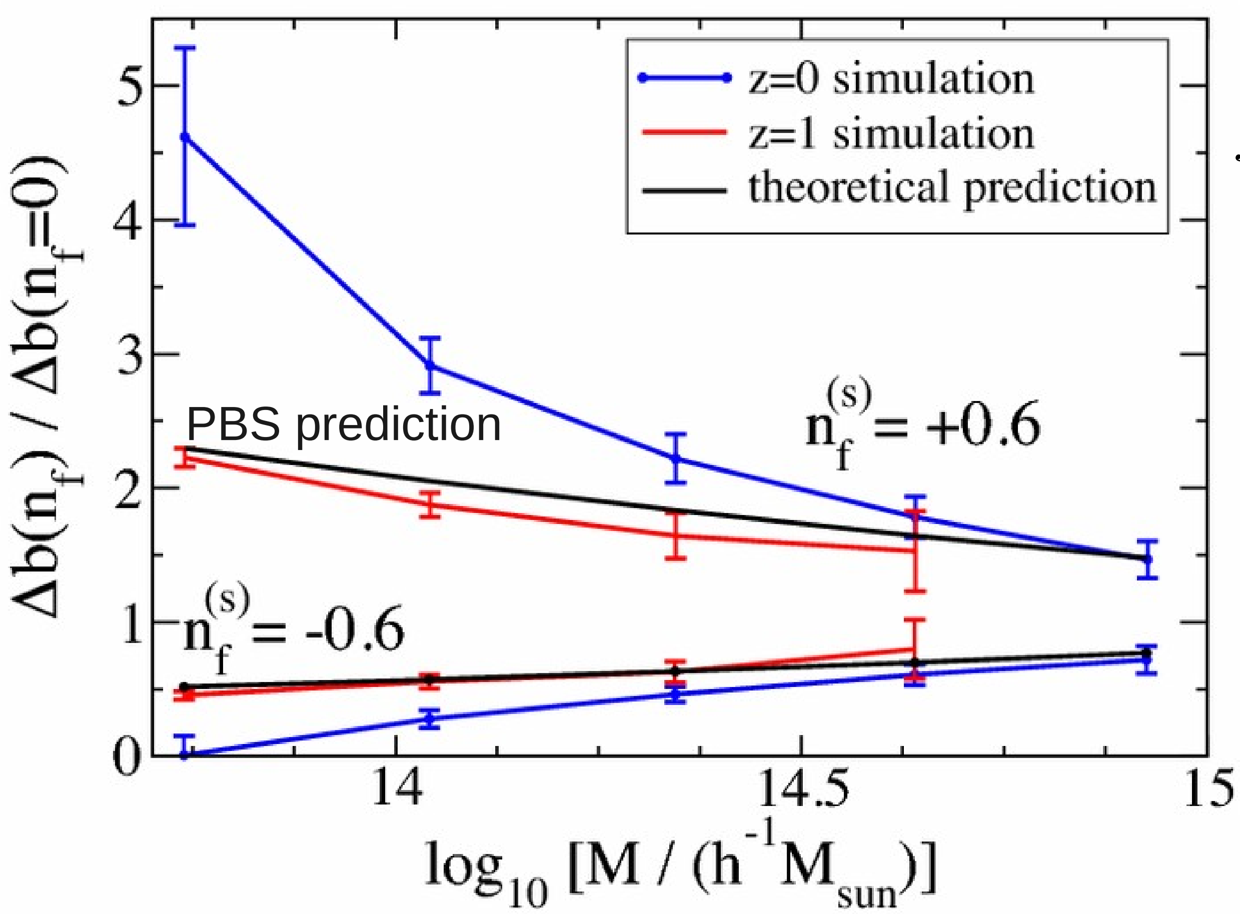
For well-resolved halos and SO finder...

Note: this rules out local bias as model of NG galaxy clustering

Hamaus et al., 1104.2321

II. Comparison with N-body results

- Scale-dependent local model … not !

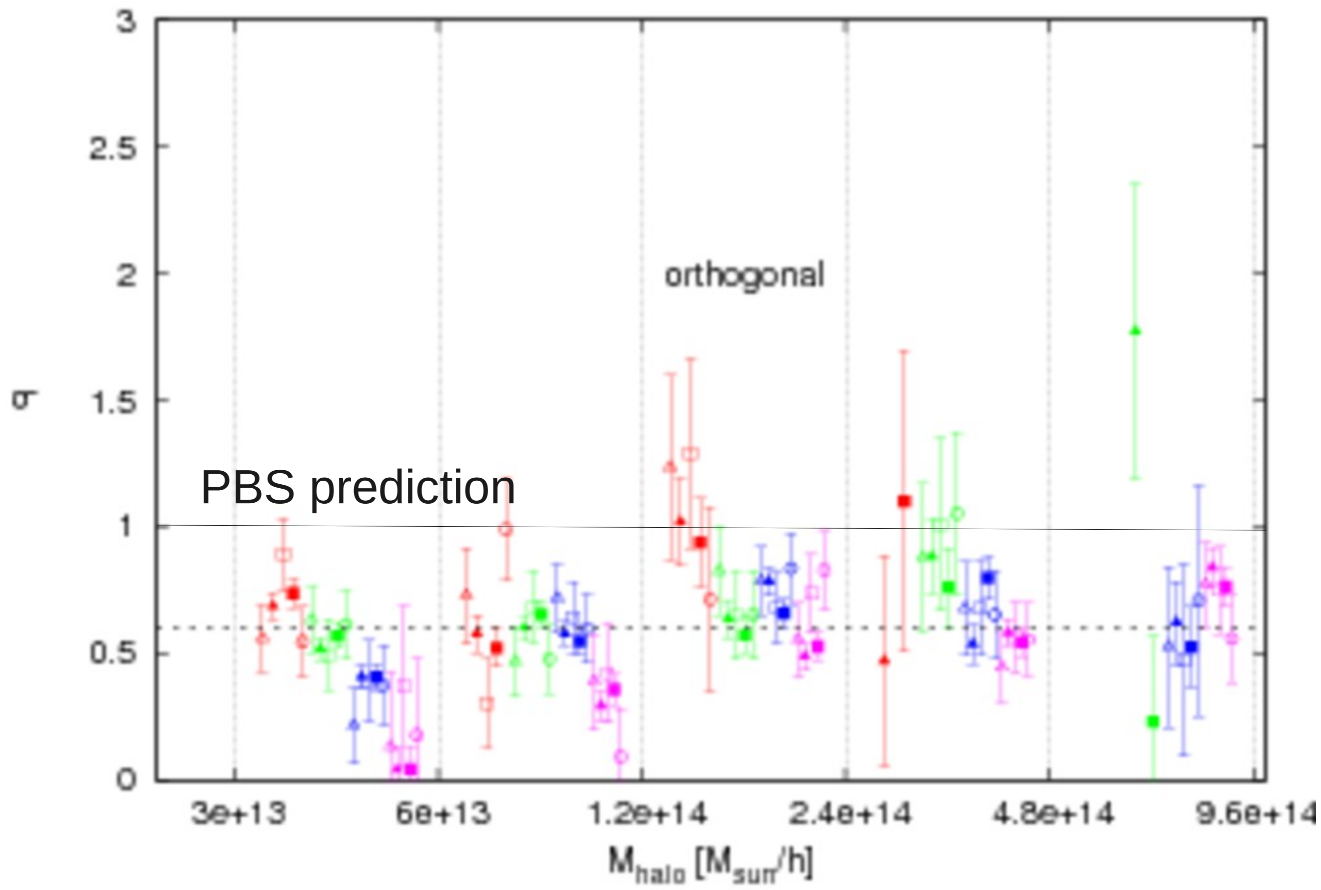


$$f_{\rm NL}(k) = f_{\rm NL,p} \left(\frac{k}{k_p} \right)^{n_f}$$

Shandera et al, 1010.3722

II. Comparison with N-body results

- Folded/orthogonal model … also not !



Wagner & Verde, 1102.3229

Comparison with N-body results

- Amplitude of non-Gaussian Δb from PBS/high-peak off for *all* models beyond scale-independent local
- Scaling with *k* consistent with prediction
- What are we missing ?

Back to universal mass function

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  - More precisely, $\frac{\hat{J}}{J} = 1 + 2\left[\frac{\partial \ln \sigma^2_{\omega M}(k)}{\partial \ln \sigma^2_M} - 1\right] \phi_l(k)$
  - $\hat{J} = J$ only for the *local quadratic case*

Contribution to NG bias

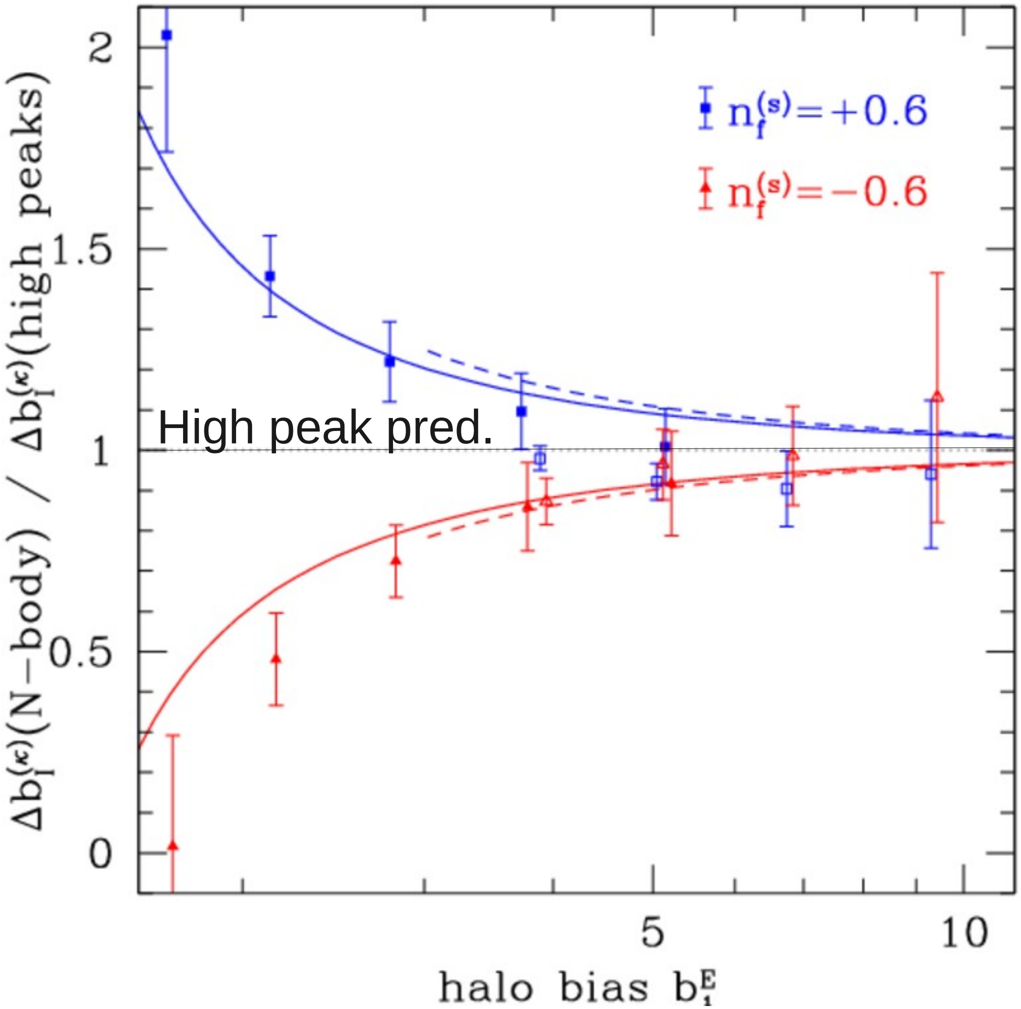
- Since $n_h \propto f(\nu)\, J$, the NG bias correction becomes

$$\Delta b(k) = 2 f_{\rm NL} \mathcal{M}^{-1}(k) \frac{\sigma^2_{\omega M}(k)}{\sigma^2_M} \left[b_1 \delta_c + 2\varepsilon_{\omega M}(k) \right]$$

$$\varepsilon_{\omega M}(k) \equiv \frac{\partial \ln \sigma^2_{\omega M}(k)}{\partial \ln \sigma^2_M} - 1$$

Updated PBS predictions

- Scale-dependent local model

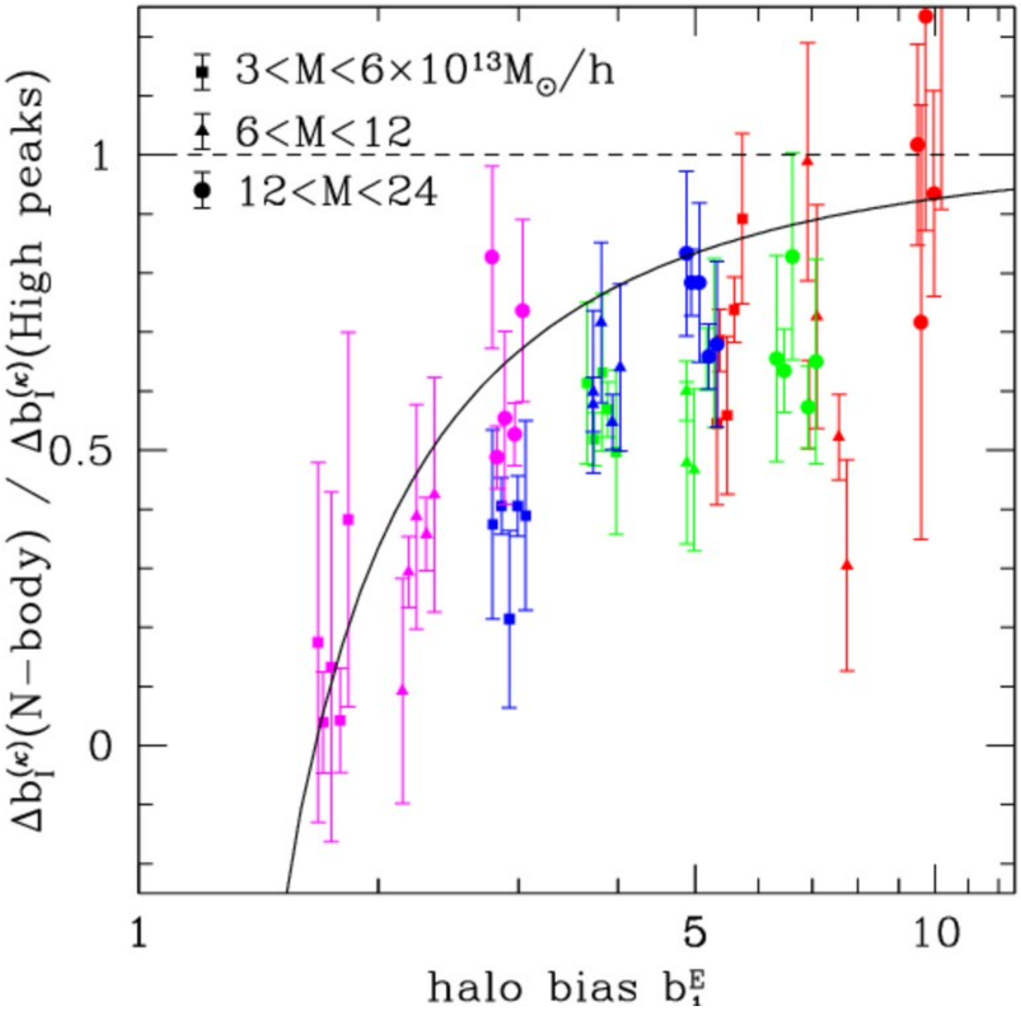


Ratio of simulations / new predictions to previous PBS/high peak prediction

Desjacques, Jeong, FS, in prep

Updated PBS predictions

- Orthogonal model



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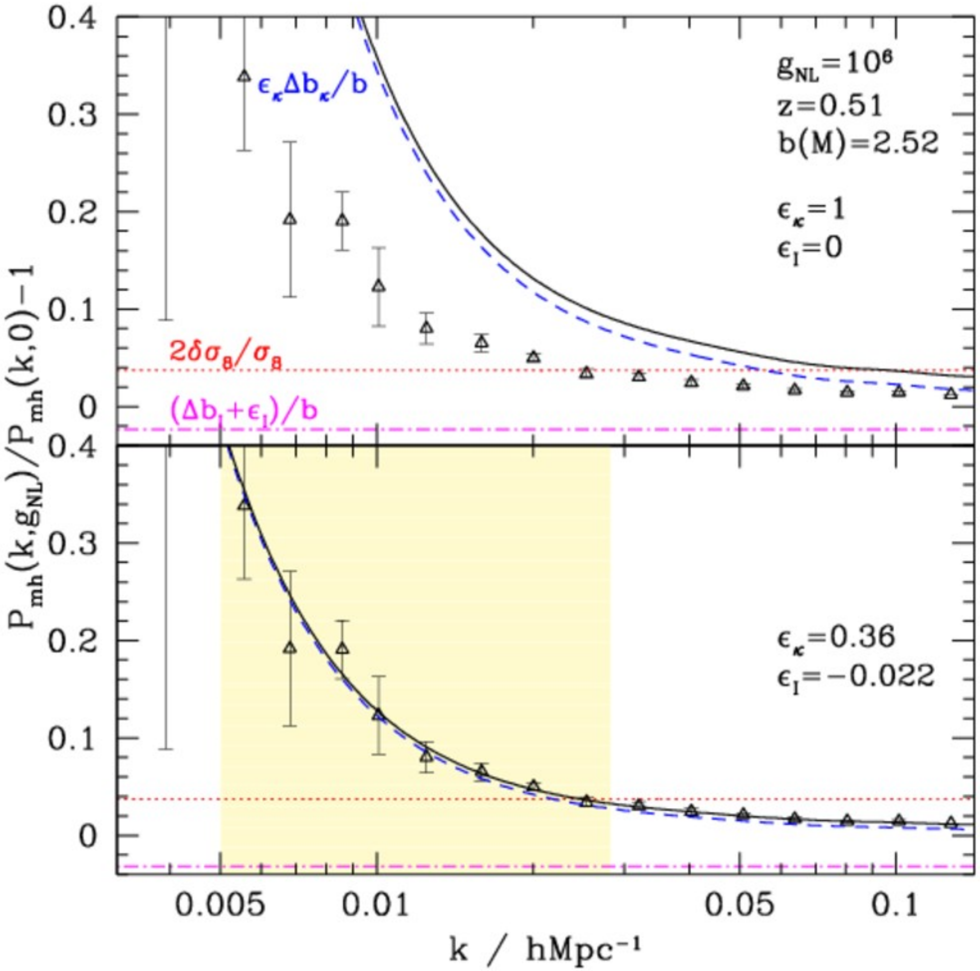
Desjacques, Jeong, FS, in prep

III. Higher-order NG

- *Cubic* local model: $\hat{\phi}(\vec{x}) = \phi(\vec{x}) + g_{\rm NL}\phi^3(\vec{x})$



PBS/high-pk in trouble again

(local biasing inconsistent as well)

Desjacques & Seljak, 0907.2257

PBS for higher-order NG

- NG with primordial *N*-point function
  - Long wavelength ϕ perturbation induces a *(N-1)*-th moment in small-scale density

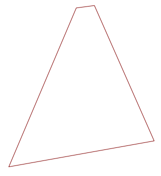
PBS for higher-order NG

- NG with primordial *N*-point function
  - Long wavelength ϕ perturbation induces a *(N-1)*-th moment in small-scale density

- General cubic model (primordial trispectrum)
  - Small-scale *skewness* induced by ϕ_l :

$$\hat{S}^{(3)}_M = 3g_{\rm NL}\,\phi_l(k)\,S^{(3)}_{\omega M}(k)$$

$$S^{(3)}_{\omega M}(k) \propto \int d^3k_1 \int d^3k_2\, \frac{T_\phi(k, k_1, k_2, |k+k_1+k_2|)}{P\,P_1\,P_2 + \ldots}$$



Example: Cubic Local NG

- Apply long/short wavelength split (f_{NL}=0 here)

$$\hat{\phi} = \hat{\phi}_l + \hat{\phi}_s = \phi + g_{\rm NL}\phi^3$$

$$\Rightarrow \hat{\phi}_s = \phi_s + (3 g_{\rm NL} \phi_l)\phi_s^2$$

Keeping only mixed terms

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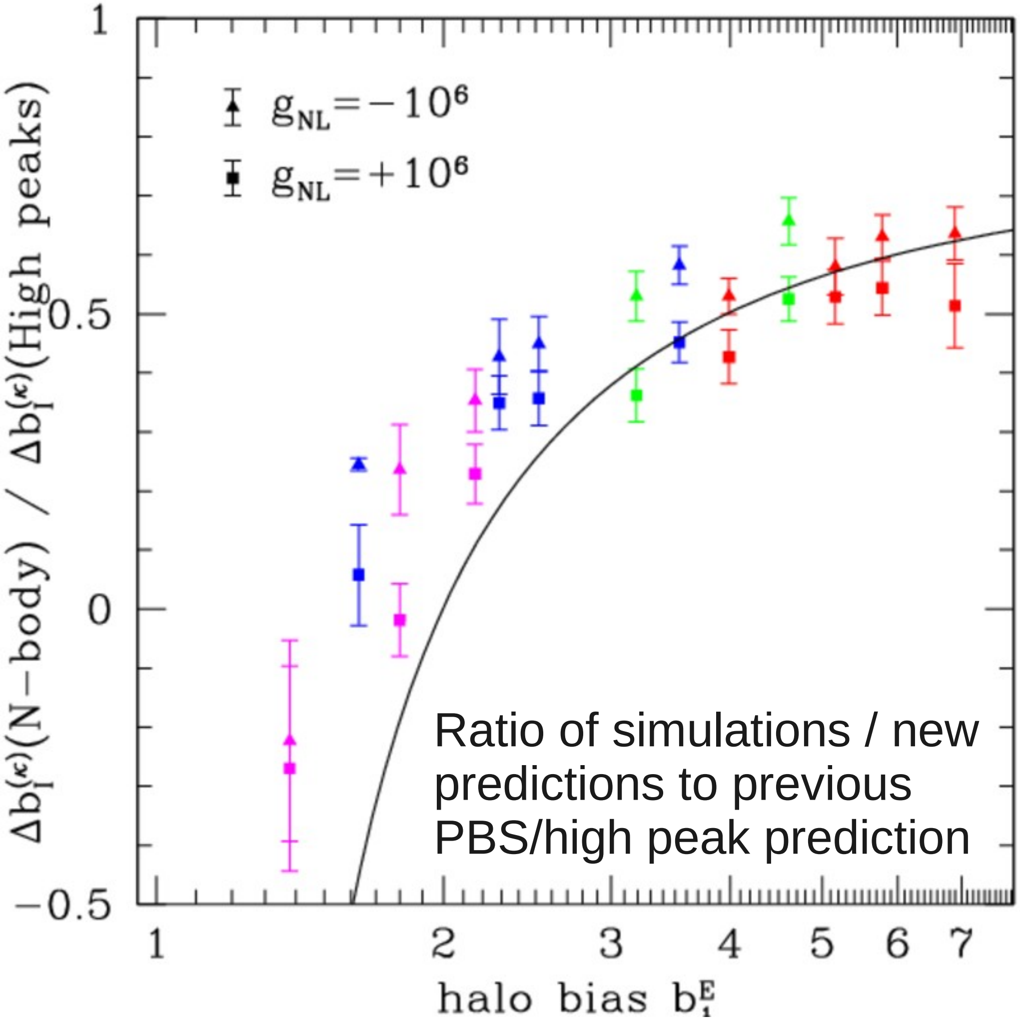
Keeping only mixed terms

- In Universe with local g_{NL}, a region with uniform ϕ_l looks like Universe with *effective local* $f_{\rm NL} = 3g_{\rm NL}\phi_l$
  - *Mass function in f_{NL}* models tells us about *bias in g_{NL}* models !

Updated predictions for g_{NL}

- Cubic local model



Here, evaluated $\partial \ln n_h / \partial S_M^{(3)}$

using Edgeworth expansion.

Result contains term

$\propto \partial \ln S_M^{(3)} / \partial \ln \sigma_M$

from mapping $d\nu \to dM$

Desjacques, Jeong, FS, in prep

Summary

- Non-Gaussian halo bias on large scales now understood
  - in the universal mass function picture

See also talks by Jeong, Yoo, Desjacques

- Likely accurate to ~10% in Δb (and f_{NL}) for virialized halos
- Next steps:
  - Corrections to b2, … ($\rightarrow$ bispectrum)
  - Is universal mass function ansatz good enough ? What about smaller scales ?

Aspen Winter Conference Jan 30 – Feb 5, 2012

- "Inflationary theory and its confrontation with data in the Planck era"

Organizers / contact:

Dore
Schmidt
Senatore
Smith

Δb in cubic local model

- $$\Delta b(k) = \frac{\partial \ln \hat{n}}{\partial \ln \hat{\sigma}_R} \frac{\partial \ln \hat{\sigma}_R}{\partial \delta_l(k)} + \frac{\partial \ln \hat{n}}{\partial \hat{S}_R^{(3)}} \frac{\partial \hat{S}_R^{(3)}}{\partial \delta_L(k)} + \ldots$$

- Most common approach: Edgeworth expansion around Gaussian mass function

$$\Rightarrow \Delta b(k) = \left[2 f_{\rm NL} b_1 \delta_c + \frac{1}{2} g_{\rm NL} \sigma_R^2 S_{R,\rm loc}^{(3)} \epsilon_S\right] \mathcal{M}^{-1}(k)$$

$$\epsilon_S = b_2 \delta_c + \left(1 + \frac{\partial \ln S_{R,\rm loc}^{(3)}}{\partial \ln \sigma_R}\right) b_1$$

Mapping ν to M